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Naval Air Development Center
Warminster, Pennsylvania

27 August 1962

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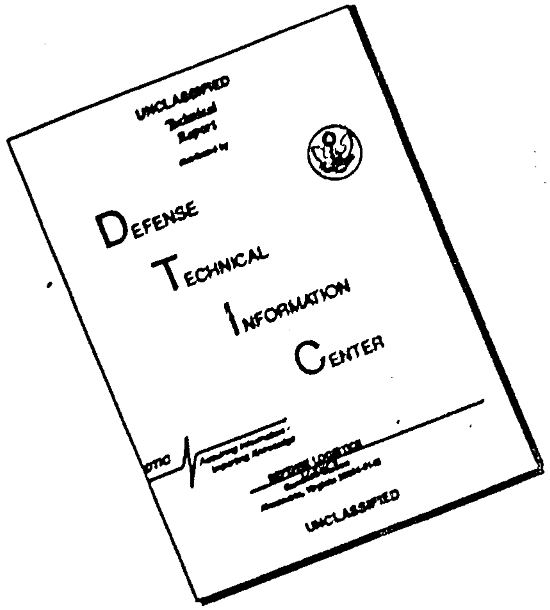

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U. S. NAVAL AIR DEVELOPMENT CENTER

JOHNSVILLE, PENNSYLVANIA

ENGINEERING DEVELOPMENT LABORATORY

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AN ANALYTICAL METHOD FOR ESTIMATING LIMITING VELOCITIES OF CAPTURED AIR BUBBLE VEHICLES AS A FUNCTION OF SEA STATE

BUREAU OF NAVAL WEAPONS
WEPTASK NO. R360FR102/2021/R01101001-6101-A

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U. S. NAVAL AIR DEVELOPMENT CENTER
ENGINEERING DEVELOPMENT DEPARTMENT
JOHNSVILLE, PENNSYLVANIA

REPORT NO. NADC-ED-L6269

Subj: WEPTASK No. R360FR102/2021/R01101001-6101-A; An Analytical Method for Estimating Limiting Velocities of Captured Air Bubble Vehicles as a Function of Sea State

Encl: (1) List of Symbols
(2) Figures 1 through 6

1. The CAB vehicle achieves lift in a manner similar to that of a planing surface but with one major exception. The boat rides on a volume of air trapped between its bottom and the water surface by sideboards on the sides and movable seals (small planing skis) fore and aft. In smooth water operation the skis do not have to move up and down to retain the trapped air and the volume of trapped air remains constant. However, when the vehicle is traveling over waves the situation becomes quite complex. This report attempts to determine the limitations resulting from operation over waves by considering several idealized modes of operation. The results of the analysis indicated that limitations on maximum g forces and fan horsepower resulted in limiting vehicle velocity as a function of wave height. Because of the assumptions made and the idealized nature of the analysis (which includes the use of sinusoidal waves, which is somewhat removed from the confused seas generally found in nature) application of the results obtained herein to a practical vehicle should be done only with full knowledge of these assumptions and their limitations.

2. Three modes of operation were considered. Common assumptions to all three modes were:

(1) Sinusoidal waves

(2) The air in the CAB vehicle bubble is incompressible.

(3) The motion of CAB vehicle in the horizontal plane is perpendicular to the wave front.

(4) The fore and aft skis are ideally movable up and down so that they maintain perfect contact with the wave surface.

(5) The vertical movement of the skis was long enough to prevent the crest of the waves from touching the bottom of the boat.

(6) The ratio of ski length to wavelength is sufficiently small so that the skis do not interfere with the nature of the wave passing into the bubble region.

(7) The wave height (trough to crest) is equal to 1/20 of the wavelength for the examples calculated in figures 2, 3 and 5, although the equations derived do not contain this assumption.

To graphically illustrate the application of the equations derived, a CAB vehicle of the following dimensions will be used:

Weight: 300 long tons
Length: 142 feet
Beam 47 feet
Specific Loading: 100 p.s.f.
Fan Horsepower: 900 (20% IPH)
Installed Propulsive Horsepower: 4500

3. Mode 1: Constant Bubble Volume Mode: In this mode of operation the CAB vehicle remains parallel to the undisturbed water surface so that passage over waves results in a heaving motion only. In order to retain the air within the bubble it is assumed that the skis are ideally movable. In order to calculate the vertical acceleration resulting from this mode of operation, it is first necessary to calculate the change in the volume of the CAB vehicle bubble resulting from the CAB vehicle operating at a fixed height (from the undisturbed surface) over sinusoidal waves as in figure 1. It is found that this volume variation is a sinusoidal function of time. In this constant bubble volume mode it is desired that the bubble volume have no variation. To accomplish this zero volume variation, the CAB deck moves up and down sinusoidally in an appropriate way. This, in turn, results in sinusoidal acceleration of the CAB vehicle deck. Physically this can be represented by assuming the volume variation as a function of time (as obtained by permitting the CAB vehicle to operate at a fixed height above the undisturbed water surface) is contained within a cylinder which is fitted with a piston at one end. If the area of the piston is equal to the planform area of the CAB vehicle, the resulting displacement and acceleration of the piston will be equal to that of the constant volume CAB vehicle operating over a wavy surface.

The instantaneous value of h in figure 1 is:

$$h = h_o - \frac{H}{2} \sin \frac{2\pi x}{\lambda} \qquad (1)$$

$$V = B \int_{x_1}^{x_2} h\,dx \qquad \text{or} \tag{2}$$

$$V = B\, h_o (x_2 - x_1) - \frac{\lambda H}{4\pi} \left(- \cos \frac{2\pi x_2}{\lambda} + \cos \frac{2\pi x_1}{\lambda}\right) \tag{3}$$

From trigonometric substitution for a difference of cosines:

$$V = B\, h_o (x_2 - x_1) - \frac{\lambda H\,2}{4\pi} \sin \frac{2\pi}{2\lambda} (x_2 + x_1) \sin \frac{2\pi}{2\lambda} (x_2 - x_1) \tag{4}$$

Let:

$$x_1 = V_r t_1 \tag{5}$$

$$x_2 = V_r t_1 + L \tag{6}$$

Substituting equations (5) and (6) for x_1 and x_2 into equation (4):

$$V = B\, h_o L - \frac{\lambda H}{2\pi} \sin \frac{\pi}{\lambda} (2V_r t + L) \sin \frac{\pi L}{\lambda} \tag{7}$$

The above equation gives the volume as a function of time. The significance of the sin $\frac{\pi L}{\lambda}$ term is explained in paragraph 6. If the volume variation of equation (7) is permitted to act within a cylinder which is fitted with a piston whose area is equal to the product of LB, the resultant displacement of the piston will be that of the constant volume CAB vehicle traveling over a wavy surface or:

$$s = \frac{V}{LB} \quad \text{and} \quad \ddot{s} = \frac{\ddot{V}}{LB} \tag{8}$$

The second derivative of equation (7) with respect to time is:

$$\ddot{V} = \frac{2\pi B H V_r^2}{\lambda} \sin \frac{\pi}{\lambda} (2V_r t + L) \sin \frac{\pi L}{\lambda} \tag{9}$$

Dividing by the CAB planform surface area LB:

$$\ddot{s} = \frac{2\pi H V_r^2}{\lambda L} \sin \frac{\pi}{\lambda} (2V_r t + L) \sin \frac{\pi L}{\lambda} \tag{10}$$

Solving for V_r for maximum value of $\ddot{s}$, which is $\ddot{S}$

$$V_r = \sqrt{\frac{\lambda L \ddot{S}}{2\pi H \sin \frac{\pi L}{\lambda}}} \tag{11}$$

Figure 2 shows a plot of equation (11) for the 142 foot CAB for a value of $\ddot{S}$ equal to 1/4 g.

4. Mode 2: Monorail Mode: This mode of operation assumes that the CAB vehicle operates at a fixed height above the undisturbed water surface by adjusting the volume of air in the bubble to compensate for the change in volume brought about by passage over the wavy surface. Thus the vehicle operates as if suspended by a monorail and experiences no pitching or heaving accelerations. In an actual vehicle this can be accomplished by dumping air overboard when the volume is decreasing and resupplying air by use of a fan when the volume is increasing. Determination of the instantaneous air rate in cubic feet per second that must be alternately supplied and removed from the air bubble can be found by taking the first derivative of equation (7).

$$q = \frac{dV}{dt} = BHV_r \cos \frac{\pi}{\lambda}(2V_r t + L) \sin \frac{\pi L}{\lambda} \tag{12}$$

If it is assumed the fan must supply air at a rate equal to the maximum instantaneous value indicated by equation (12), then q equals:

$$Q = BHV_r \sin \frac{\pi L}{\lambda} \tag{13}$$

The limiting wave height as a function of fan horsepower is:

$$H = \frac{550\ P}{\Delta p B V_r \sin \frac{\pi L}{\lambda}} \quad \text{or} \quad V_r = \frac{550\ P}{\Delta p B H \sin \frac{\pi L}{\lambda}} \tag{14}$$

Figure 3 shows a plot of limiting velocity vs. wavelength (eq. 14) for the 142 foot CAB vehicle. For large wavelengths the $\sin \frac{\pi L}{\lambda}$ term in eq. 14 approaches $\frac{\pi L}{\lambda}$ and upon appropriate substitution H will cancel with λ. Thus the limiting velocity will become independent of λ and approaches a constant as is shown in eq. 13.

A more optimistic analysis considers the average horsepower in lieu of the maximum value. Since the average value of a sine function is equal to the maximum value divided by $\frac{2}{\pi}$, the average fan horsepower will be $P_{avg} = \frac{P_{max}}{\pi}$ or the maximum wave height will be: .

$$H = \pi \left(\frac{550\ P}{\Delta p B V_r \sin \frac{\pi L}{\lambda}} \right) \tag{15}$$

5. Mode 3: Combination Mode: This mode of operation combines the first and second modes. In effect, this mode of operation reduces the heaving amplitude of constant volume operation by withdrawing or supplying air to the CAB vehicle bubble region at the proper time. The greater the quantity of air the greater the reduction in amplitude and acceleration

until a point is reached where the acceleration is reduced to zero. In the mode of operation considered here, rather than use the air supply to reduce the acceleration to zero for a given wave height and limiting velocity (as was done in Mode 2), the acceleration is maintained at a constant value and the limiting velocity is increased for a given wave height. A model that represents the mathematical method used is shown in figure 4. The flow output from the first cylinder represents the flow variation resulting from passage of the CAB over the wavy surface as is defined by equation (12). The flow output of the third cylinder represents the additional air to be alternately added and withdrawn. The mathematical expression for q_{add} as a function of time is selected to optimize the utilization of the available air supply. The magnitude of the quantity of the available air supply is determined by the fan horsepower. The motion of the piston of the second cylinder is the resultant of the predetermined output of the first and third cylinders and will be representative of the motion of the deck of the CAB vehicle with additional air supply if the piston area is equal to the surface area of the CAB vehicle deck (L x B). Expressed mathematically where q represents the instantaneous flow rate:

$$q_{res} = q_{cab} + q_{add} \tag{16}$$

Utilization of the supply air is maximized if it is of the same function of time as the flow rate resulting from passage of the CAB vehicle over the wavy surface, but 180° out of phase:

$$q_{add} = -Q \cos \frac{\pi}{\lambda}(2V_r t + L) \qquad \text{or} \tag{17}$$

$$q_{add} = -\frac{550P}{\Delta p} \cos \frac{\pi}{\lambda}(2V_r t + L) \tag{18}$$

Substituting equation (8) and (14) into equation (12):

$$q_{res} = (BHV_r \sin \frac{\pi L}{\lambda} - \frac{550P}{\Delta p}) \cos \frac{\pi}{\lambda}(2V_r t + L) \tag{19}$$

Taking the derivative of eq. 15 and dividing by L x B:

$$\ddot{s} = \frac{2\pi V_r}{LB\lambda}(BHV_r \sin \frac{\pi L}{\lambda} - \frac{550P}{\Delta p}) \sin \frac{\pi}{\lambda}(2V_r t + L) \tag{20}$$

and

$$\ddot{S} = \frac{2\pi HV_r^2}{\lambda L} \sin \frac{\pi L}{\lambda} - \frac{2\pi 550PV_r}{LB\lambda\Delta p} \tag{21}$$

Solving for V_r by use of the quadratic equation:

$$V_r = \frac{550P}{2BH\Delta p \sin\frac{\pi L}{\lambda}} + \frac{1}{2}\sqrt{\frac{550P}{BH\Delta p \sin\frac{\pi L}{\lambda}} + \frac{2\lambda L\ddot{S}}{\pi H \sin\frac{\pi L}{\lambda}}} \qquad (22)$$

Equation 22 is the mathematical expression for Mode 3 operation. When P is equal to zero it reduces to an expression representing Mode 1 operation only as determined by equation (11); and when S is equal to zero it reduces to Mode 2 operation only as determined by equation (14). Thus, Modes 1 and 2 are a special case of Mode 3. Figure 5 shows a plot of equation (22) for the 142 foot CAB vehicle with $\ddot{S}$ equal to 1/4 g.

6. The following discussion is presented to provide the reader with an understanding of the geometrical significance of the mathematical expression $\sin\frac{\pi L}{\lambda}$ which appears in equation (7). The term $\sin\frac{\pi L}{\lambda}$ regulates the magnitude of the instantaneous value of volume and is dependent upon the ratio $\frac{L}{\lambda}$. A drawing showing the geometrical relation between CAB vehicle and waves for the extreme values of the $\sin\frac{\pi L}{\lambda}$ term is shown in figure 6. The shaded areas in figure 6 represent the area swept out by the vehicle motion during a short interval of time. The shaded area determined by the front ski represents the volume gained and the shaded area determined by the rear ski represents the volume lost. Figure 6a represents the CAB vehicle configuration when $\frac{L}{\lambda}$ equals one. For this case the front and rear ski contact the wavy surface so that the geometric relationship between ski and wavy surface is common to both. For values of $\frac{L}{\lambda}$ equal to one, or for any value where $\frac{L}{\lambda} = n$ and n is any integer, the shaded area representing the volume gained is exactly equal to the shaded area representing the volume lost and the net volume change is zero. Figure 6b represents the CAB vehicle configuration for the case where $\frac{L}{\lambda}$ equals $\frac{1}{2}$. For this case the front ski and rear ski are out of phase by 180° and the difference in length between the front ski and rear ski is at a maximum. Therefore, for values of $\frac{L}{\lambda}$ equal to $\frac{1}{2}$, or for any value where $\frac{L}{\lambda} = (n + \frac{1}{2})$ and n is any integer or zero, the difference between volume gained and volume lost is a maximum. Thus it can be seen that if the conditions in natural seas even approximate the symmetrical waves used in this analysis, the ratio of bubble length to wave length becomes of real significance.

It is the above phase condition for a constant length vehicle with variable wave length that is evident as sharp peaks in allowed velocities V_r in figures 2, 3 and 5.

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List of Symbols

SYMBOL	DESCRIPTION	UNIT
B	Width of CAB air bubble	ft.
h	Vertical distance between CAB deck and wave surface	ft.
H	Wave height crest to trough	ft.
L	Length of CAB vehicle air bubble	ft.
Δp	Bubble pressure	p.s.f.
q	Instantaneous air flow rate	$\frac{ft.^3}{sec.}$
Q	Maximum value of q	$\frac{ft.^3}{sec.}$
P	Horsepower	
t	time	secs.
x	Horizontal displacement	ft.
V	Volume	$ft.^3$
V_r	Relative velocity between CAB vehicle and waves	ft./sec.
s	Vertical displacement of CAB vehicle	ft.
S	Maximum value of s	ft.
λ	Wave length	ft.

Enclosure (1)

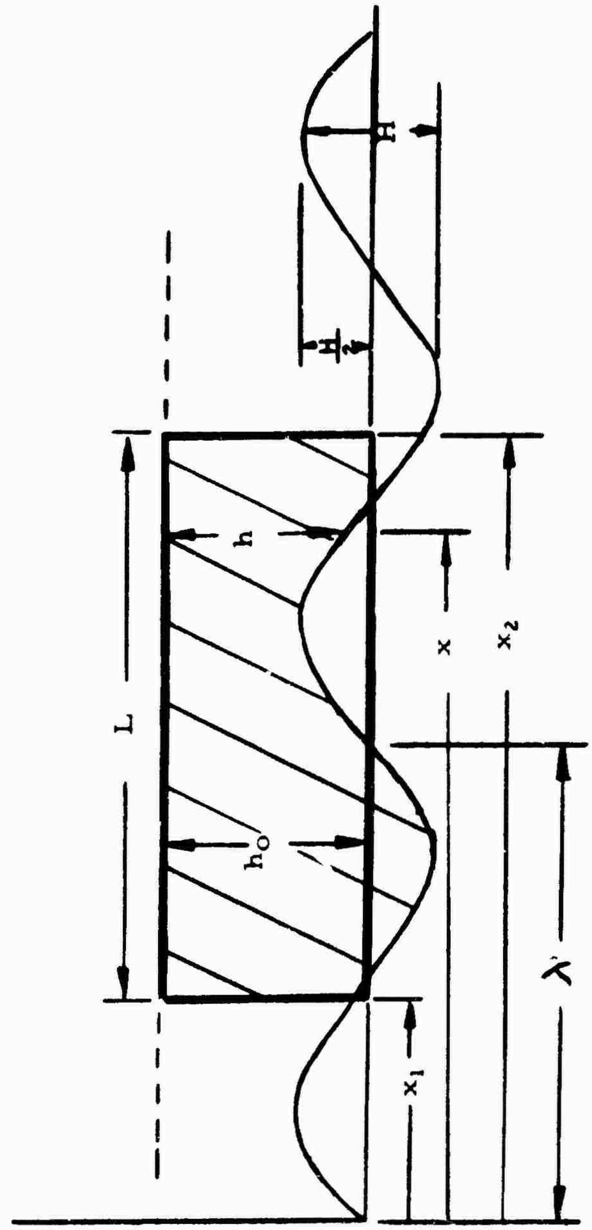


Figure 1

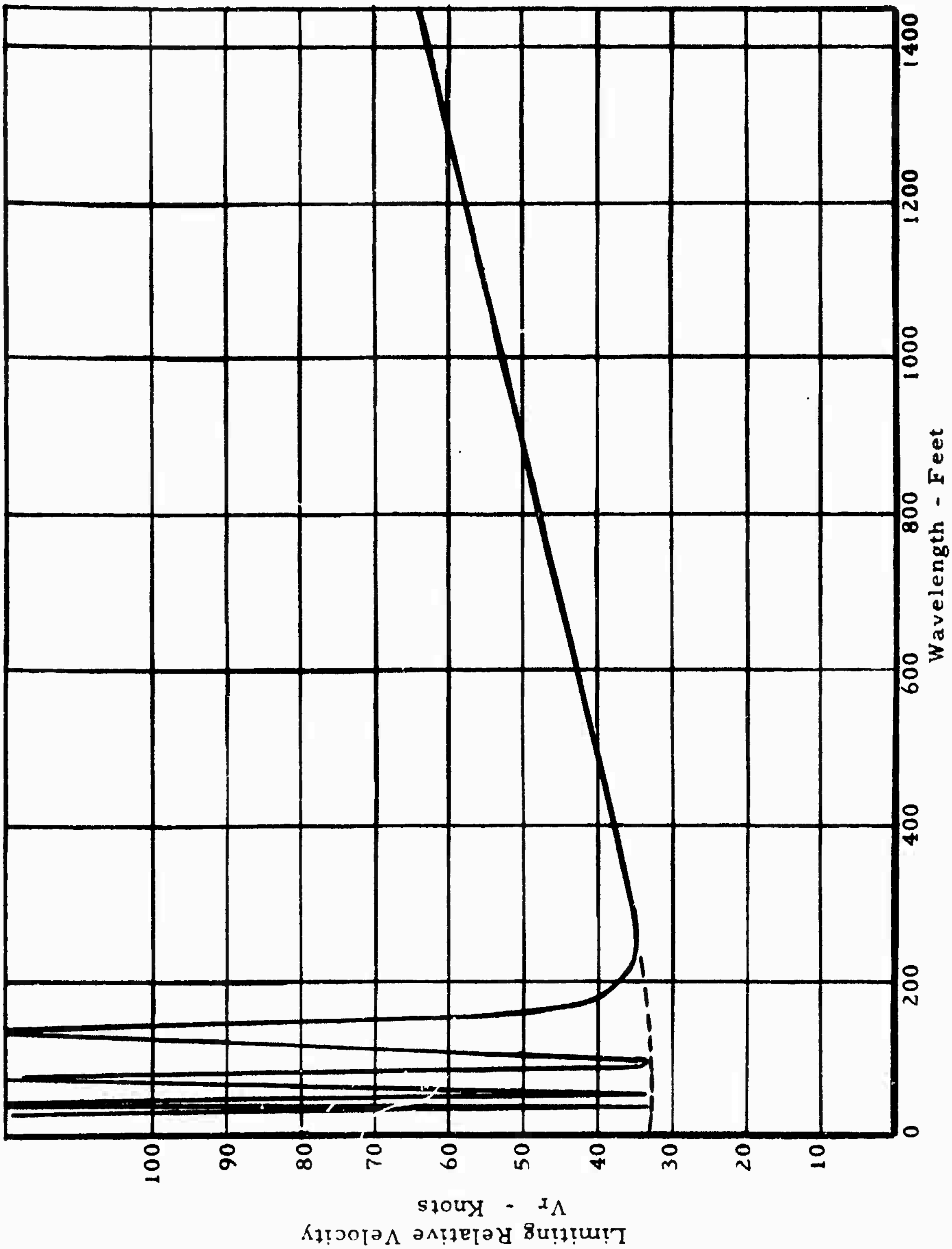


Figure 2. Mode I Operation for 142' CAB
Maximum Acceleration $\frac{1}{4}$ g

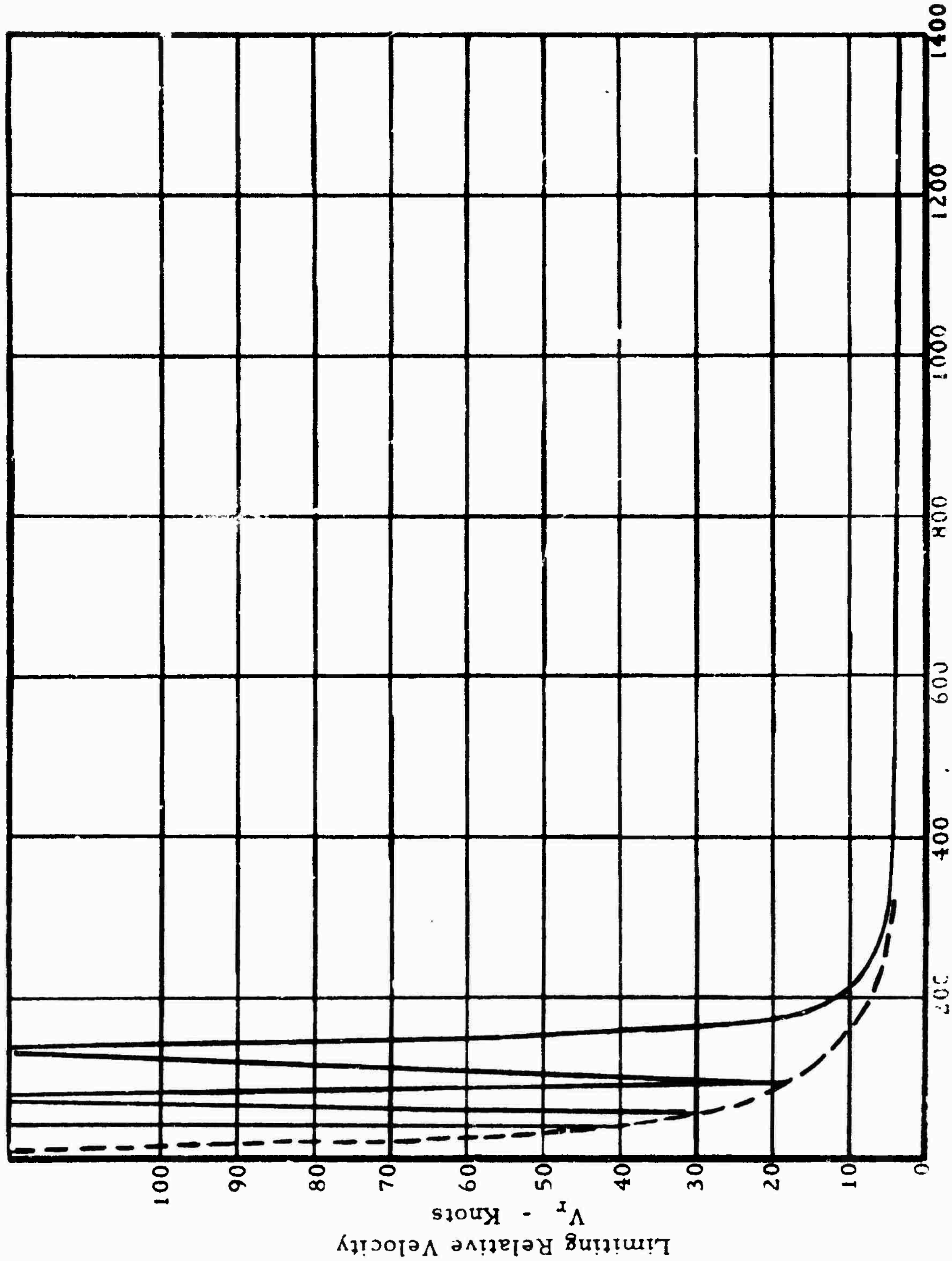
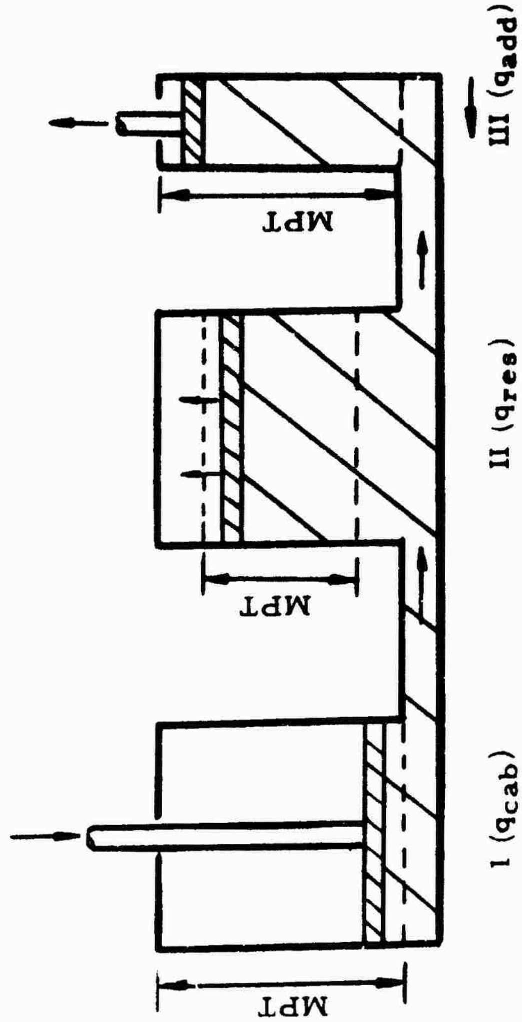


Figure 3. Mode B Operation for 14' ... dB



MPT - Maximum Piston Travel

Figure 4. Physical Equivalent of Equation 12

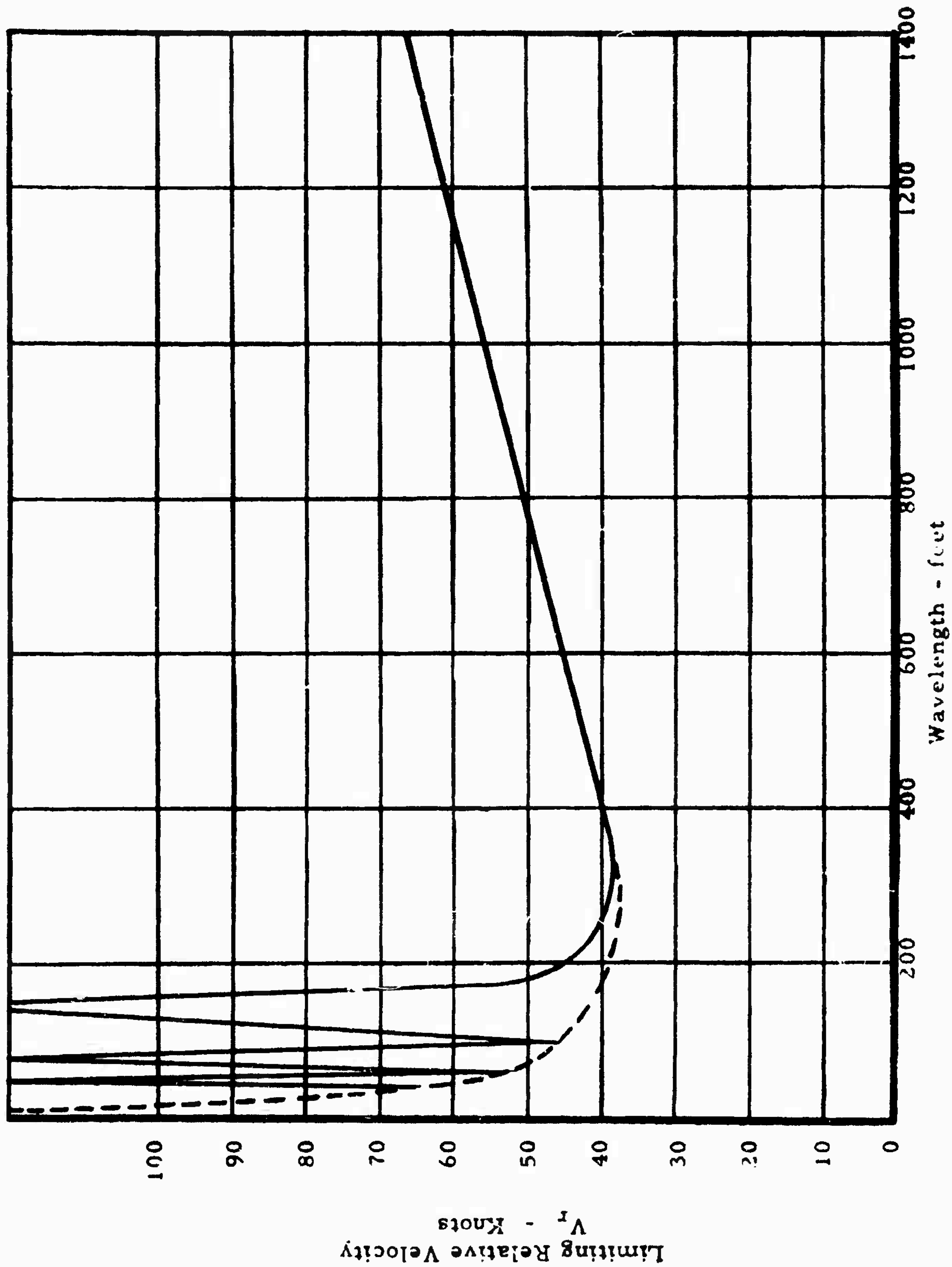
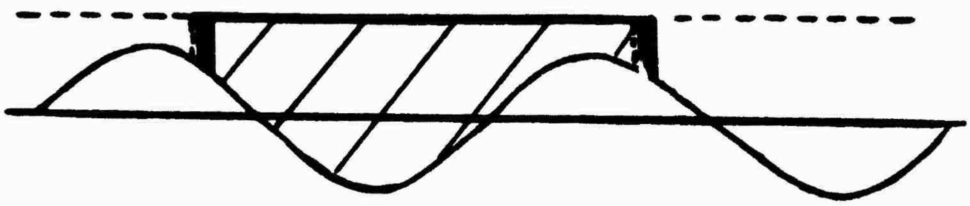
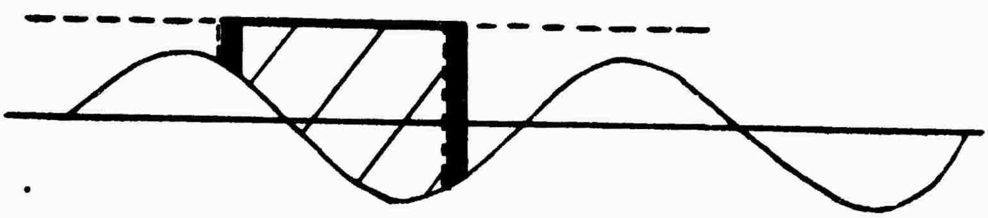


Figure 5. Mode III Operation for 142' CAB
Maximum Acceleration $\frac{1}{4}$ g



6a $\frac{L}{\lambda} = 1.0$



6b $\frac{L}{\lambda} = \frac{1}{2}$

Figure 6